Stereological Techniques for Solid Textures

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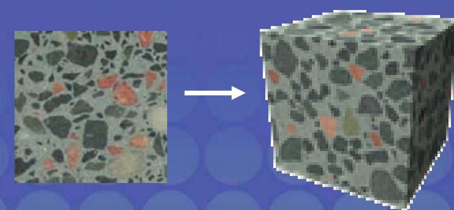
SIGGRAPH2004

Objective

Given a 2D slice through an aggregate material, create a 3D volume with a comparable appearance.



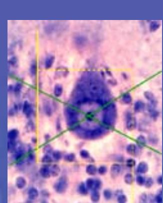
Real-World Materials

- Concrete
- Asphalt
- Terrazzo
- Igneous minerals
- Porous materials

Independently Recover…

- Particle distribution
- Color
- Residual noise

In Our Toolbox…



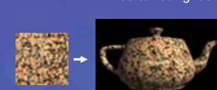
Stereology (stĕr'ē-ŏl'ə-jē)

The study of 3D properties based on 2D observations.

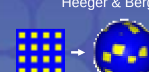
Prior Work – Texture Synthesis

- 2D→2D
- 3D→3D
- Procedural Textures
- 2D→3D
  - Heeger & Bergen 1995
  - Dischler et al. 1998
  - Wei 2003

Efros & Leung '99



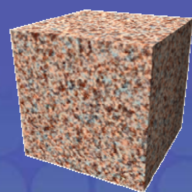
Heeger & Bergen '95



Wei 2003

Prior Work – Texture Synthesis

Input

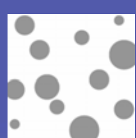


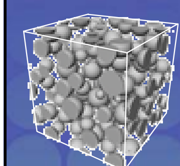
Heeger & Bergen, '95

Prior Work – Stereology

- Saltikov 1967
  Particle size distributions from section measurements
- Underwood 1970
  Quantitative Stereology
- Howard and Reed 1998
  Unbiased Stereology
- Wojnar 2002
  Stereology from one of all the possible angles

Recovering Sphere Distributions





N_A = Profile density
(number of circles per unit area)

N_V = Particle density
(number of spheres per unit volume)

$\overline{H}$ = Mean caliper particle diameter

The fundamental relationship of stereology:

$$N_A = \overline{H} N_V$$

Recovering Sphere Distributions

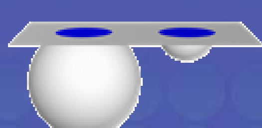
Group profiles and particles into n bins according to diameter

Particle densities = $N_A(i), \{1 \le i \le n\}$
Profile densities = $N_V(i), \{1 \le i \le n\}$

For the following examples, $n = 4$

Recovering Sphere Distributions

Note that the profile source is ambiguous



Recovering Sphere Distributions

How many profiles of the largest size?

$N_A(4)$ = K_{44} $N_V(4)$

K_{ij} = Probability that particle $N_V(j)$ exhibits profile $N_A(i)$

Recovering Sphere Distributions

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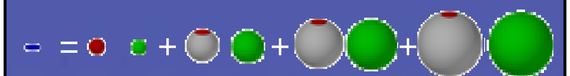
How many profiles of the smallest size?



$N_A(1) = K_{11} N_V(1) + K_{12} N_V(2) + K_{13} N_V(3) + K_{14} N_V(4)$

K_{ij} = Probability that particle $N_V(j)$ exhibits profile $N_A(i)$

Recovering Sphere Distributions

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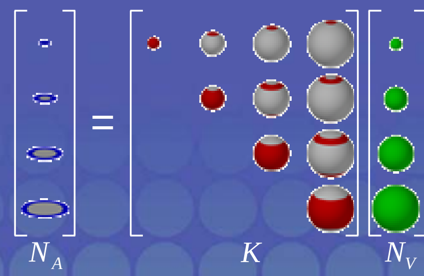
Putting it all together…



Recovering Sphere Distributions

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Some minor rearrangements…

$N_A = d_{\max} \; K \; N_V$

$d_{\max}$ = Maximum diameter

Normalize probabilities for each column j:

$$\sum_{i=1}^{n} K_{ij} = j / n$$

Recovering Sphere Distributions

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$$N_A = d_{\max} K N_V$$

K is upper-triangular and invertible

For spheres, we can solve for K analytically:

$$K_{ij} = \begin{cases} 1/n \cdot \left(\sqrt{j^2 - (i-1)^2} - \sqrt{j^2 - i^2} \right) & \text{for } j \geq i \\ 0 & \text{otherwise} \end{cases}$$

Solving for particle densities: $N_V = \frac{1}{d_{\max}} K^{-1} N_A$

Testing precision

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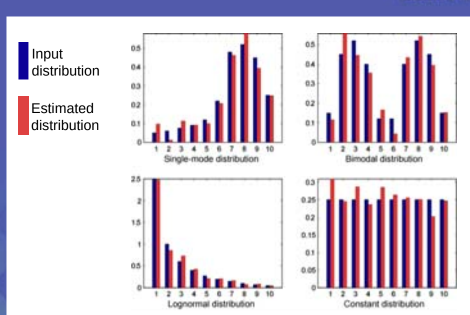

Other Particle Types

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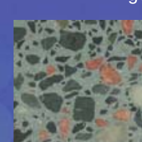
We cannot classify arbitrary particles by d/d_{max}

Instead, we choose to use $\sqrt{A / A_{\max}}$

Algorithm inputs:



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Approach: Collect statistics for 2D profiles *and* 3D particles

Profile Statistics

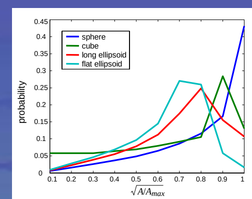
Segment input image to obtain profile densities N_A.

Input Segmentation

Bin profiles according to their area, $\sqrt{A/A_{max}}$

Particle Statistics

Look at thousands of random slices to obtain $\overline{H}$ and K



Example probabilities of $\sqrt{A/A_{max}}$ for simple particles

Recovering Particle Distributions

Just like before, $N_A = \overline{H} K N_V$

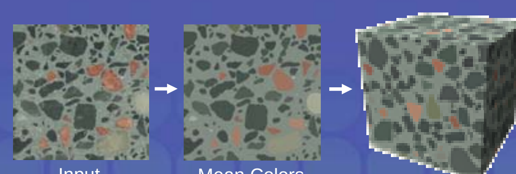
Solving for the particle densities,

$$N_V = \frac{1}{\overline{H}} K^{-1} N_A$$

Use N_V to populate a synthetic volume.

Recovering Color

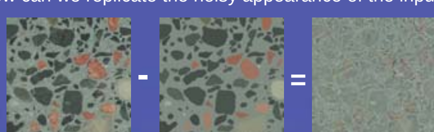
Select mean particle colors from segmented regions in the input image



Input Mean Colors Synthetic Volume

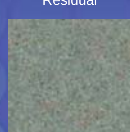
Recovering Noise

How can we replicate the noisy appearance of the input?



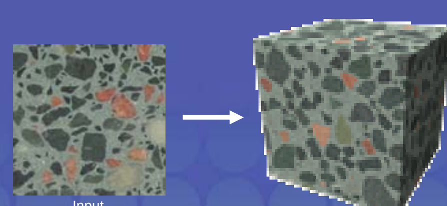
Input - Mean Colors = Residual

The noise residual is less structured and responds well to Heeger & Bergen's method



Synthesized Residual

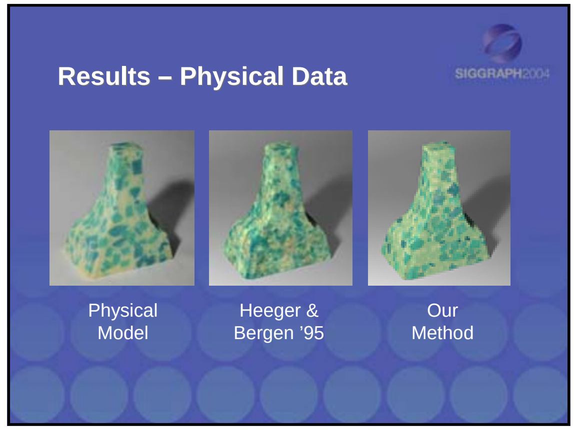
Putting it all together



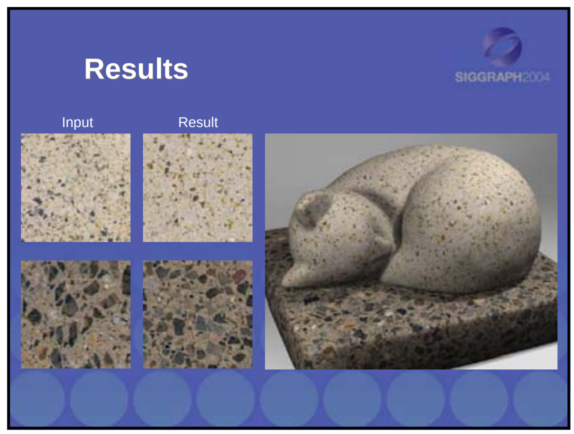
Input

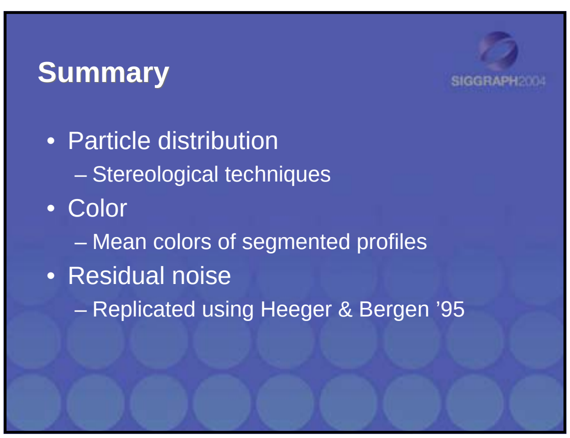
Synthetic volume with noise

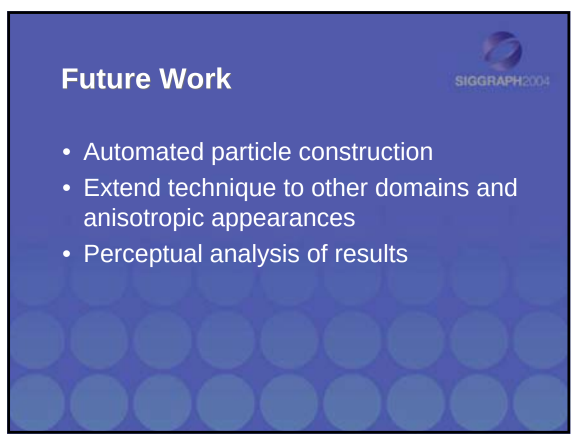

Prior Work – Revisited
SIGGRAPH2004
Input
Heeger & Bergen '95
Our result


Results – Physical Data
SIGGRAPH2004
Physical Model
Heeger & Bergen '95
Our Method


Results
SIGGRAPH2004
Input
Result


Results
SIGGRAPH2004
Input
Result


Summary
SIGGRAPH2004
• Particle distribution
– Stereological techniques
• Color
– Mean colors of segmented profiles
• Residual noise
– Replicated using Heeger & Bergen '95


Future Work
SIGGRAPH2004
• Automated particle construction
• Extend technique to other domains and anisotropic appearances
• Perceptual analysis of results

Thanks to…

SIGGRAPH2004

- Maxwell Planck, undergraduate assistant
- Virginia Bernhardt
- Bob Sumner
- John Alex